

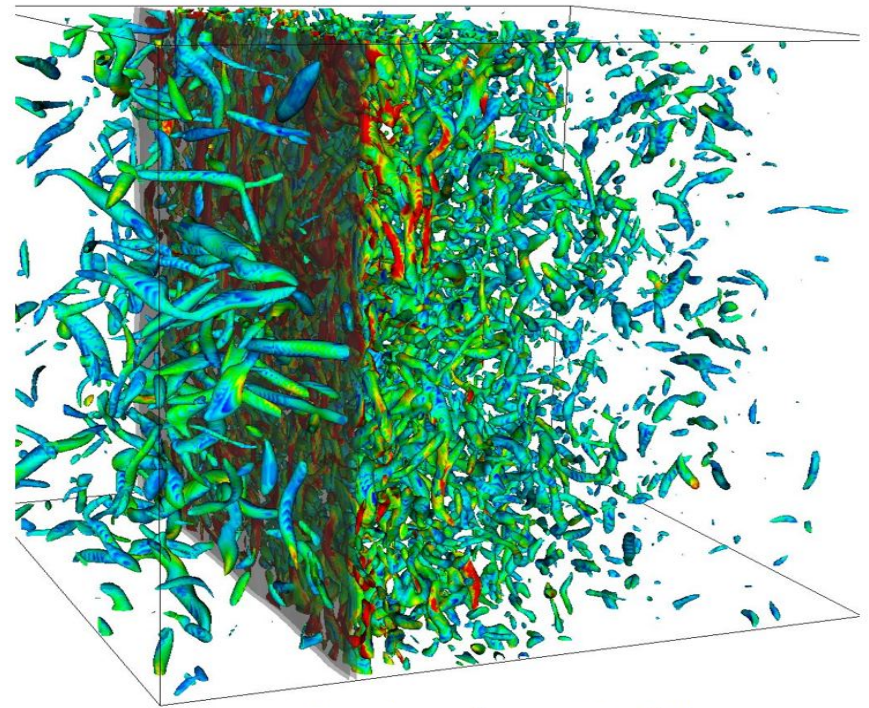
Study of Kovásznyai modes in canonical shock-turbulence interaction

Russell Quadros, Ph.D student, IIT Bombay

Yogesh Prasaad, Ph.D student, IIT Bombay

Prof. Krishnendu Sinha, IIT Bombay

Prof. Johan Larsson, University of Maryland



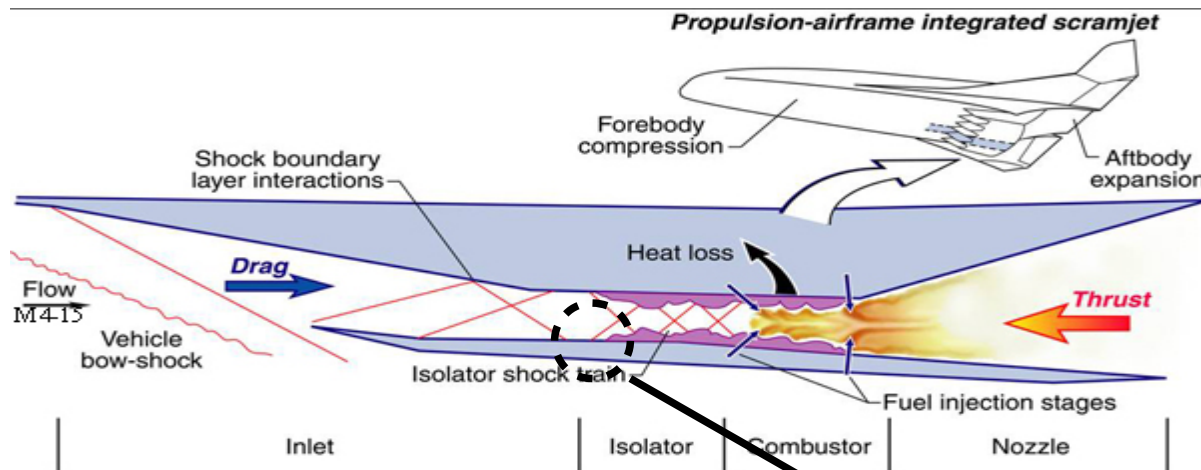
*DNS of shock turbulence interaction
by Larsson et al.*

<http://www.hypersonic-cfd.com>

<http://terpconnect.umd.edu/~jola/people.html>

Motivation

Shock/boundary-layer interaction

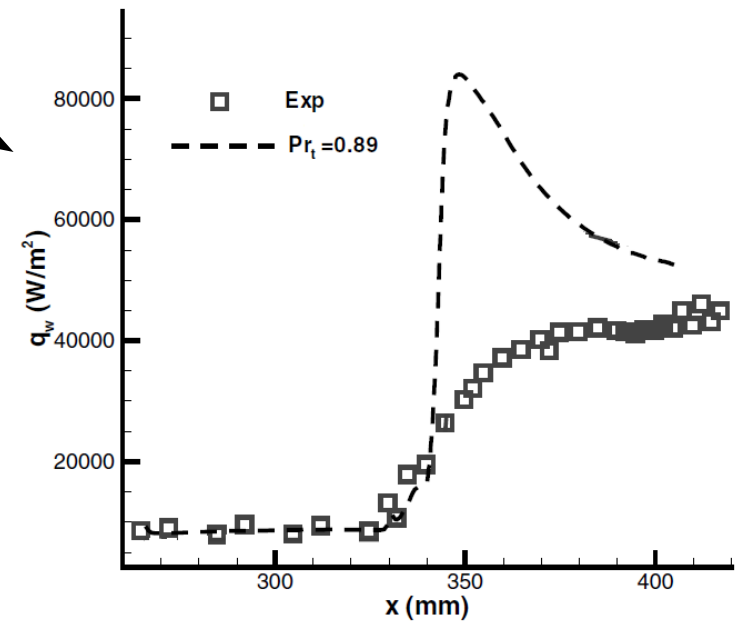


NASA Langley – Hy-V vehicle

Source: LaRC

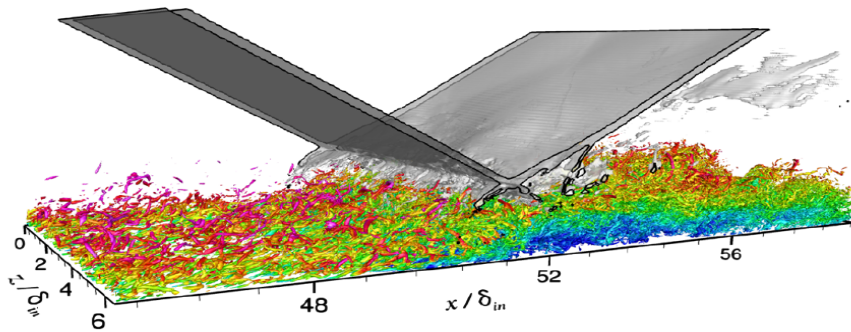
KR/LH02072001

- Shock impinging on a wall
- Interaction with a turbulent boundary layer
- High Pressure and heat loads
- RANS – heat flux prediction

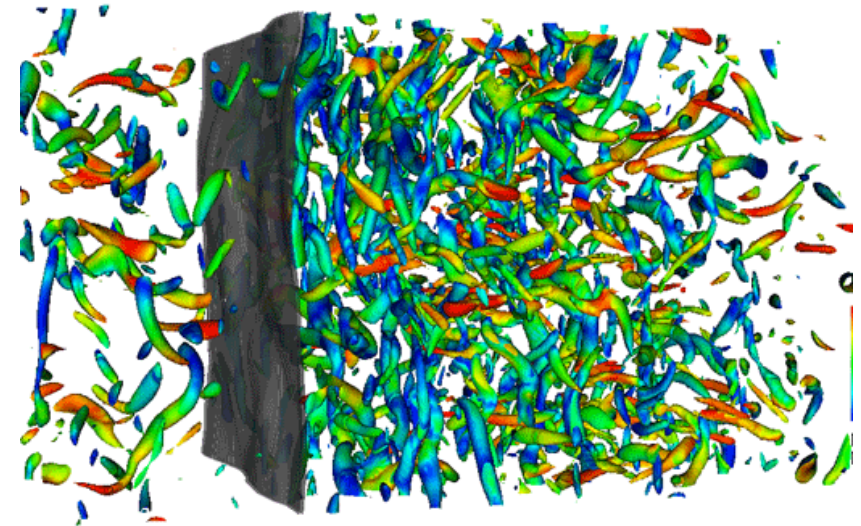


Xiao et al. (2007)

Simplified model



Pirozzoli and Bernardini (2011)



DNS by Larsson et al. 2013

Eddies extracted by the Q-criterion.

Canonical shock turbulence interaction

- Homogeneous turbulence interacting with a normal shock
- Isolates effect of shock on turbulence
- No mean gradient, separation etc.
- Parametric study possible
- Two approaches

DNS

Theory

Lee et al (1993), Mahesh et al. (1997), Wouchuk et al.(2009),
Larsson and Lele (2009), Larsson et al. (2013), Ryu and Livescu (2014)

Theory

Turbulence in supersonic flow

- Governing equations :
Linearized Navier-Stokes equations

$$\nabla \cdot \underline{v}' = -\frac{\partial}{\partial t} \left(\frac{p'}{\gamma \bar{p}} \right) + \frac{\partial s'}{\partial t}$$

$$\frac{\partial \underline{v}'}{\partial t} = -a^2 \nabla \left(\frac{p'}{\gamma \bar{p}} \right) + \nu \left(\nabla^2 \underline{v}' + \frac{1}{3} \nabla (\nabla \cdot \underline{v}') \right)$$

$$\frac{\partial s'}{\partial t} - \frac{4\nu}{3} \nabla^2 s' = \frac{4(\gamma - 1)\nu}{3} \nabla^2 \left(\frac{p'}{\gamma \bar{p}} \right)$$

$$s' = \frac{T'}{\bar{T}} - (\gamma - 1) \frac{p'}{\gamma \bar{p}}$$

Kovácsznay (1953)

- Assumptions:
- Perfect gas with constant properties
- Fluid is at rest i.e. mean velocity in the domain is zero
- Fluctuations are small compared to average values. For velocities, average speed of sound is considered.
- Only first order terms

Turbulence in supersonic flow

➤ Decoupled linearized N-S equations:

Curl of momentum 
$$\frac{\partial \underline{\omega}'}{\partial t} - \nu \nabla^2 \underline{\omega}' = 0$$

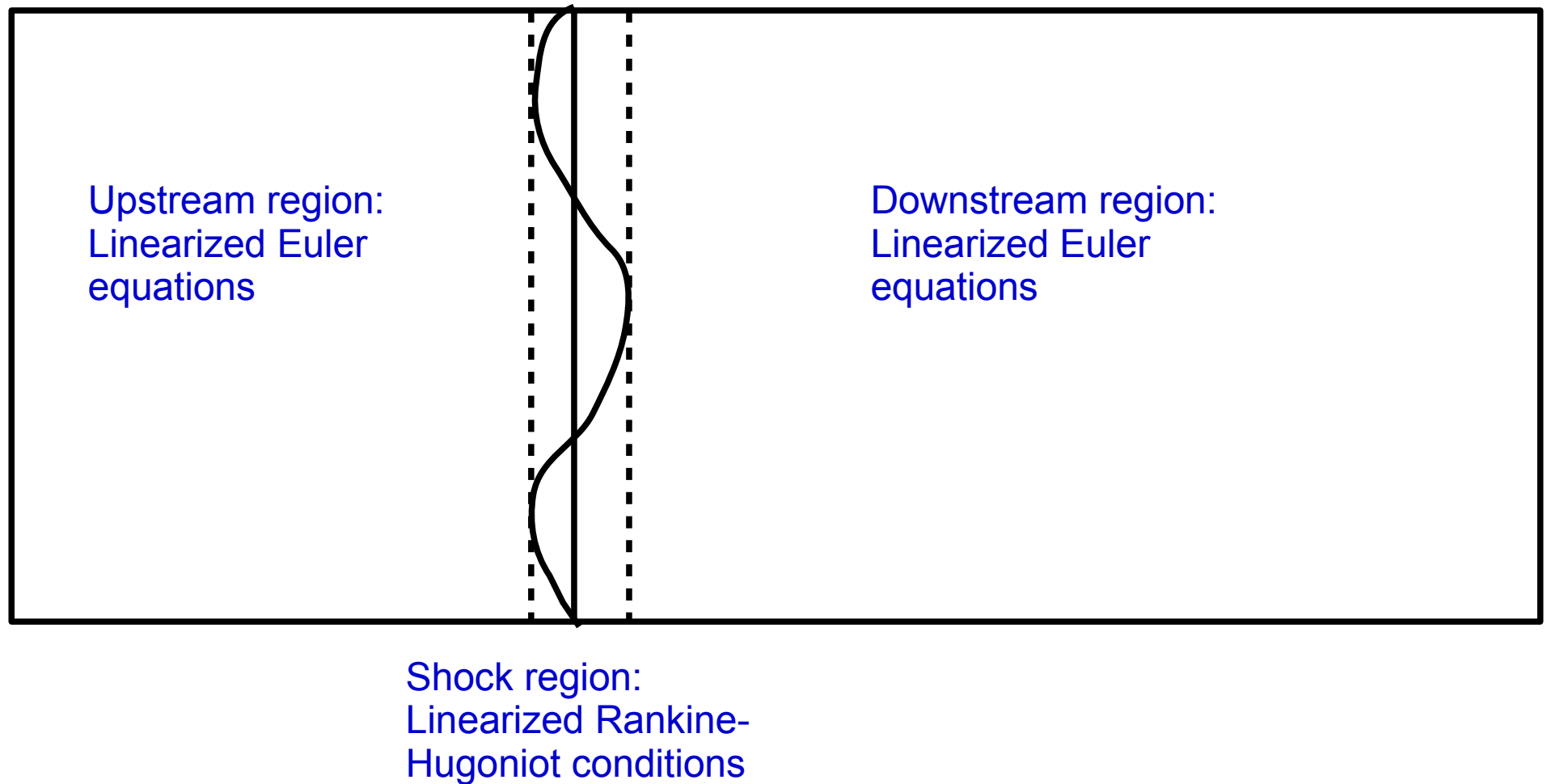
Divergence of momentum 
$$\frac{\partial}{\partial t} \left(\frac{p'}{\gamma \bar{p}} \right) - a^2 \nabla^2 \left(\frac{p'}{\gamma \bar{p}} \right) - \frac{4\nu}{3} \frac{\partial}{\partial t} \nabla^2 \left(\frac{p'}{\bar{p}} \right) = 0$$

Homogeneous solution only 
$$\frac{\partial s'}{\partial t} - \frac{4\nu}{3} \nabla^2 s' = 0$$

Kovásznay (1953)

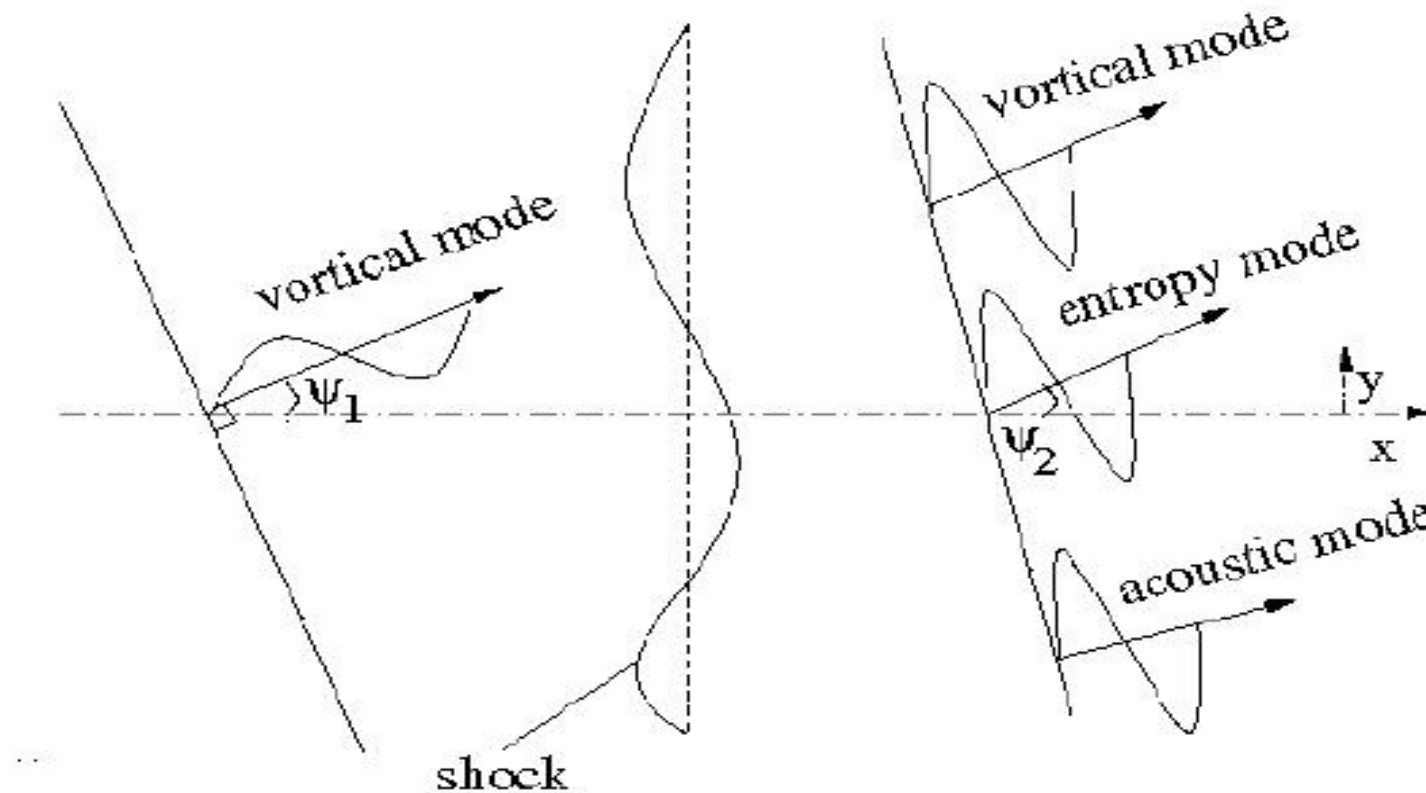
Application to shock turbulence interaction

Finite fluid velocity



Ribner (1954)

Linear interaction analysis



Upon interaction of a single mode, all three are generated and evolve independently

Ribner (1954)

Linear interaction analysis

Three independent modes exist downstream of the shock:

Vorticity - $\underline{\omega}'$

Entropy - s'

Acoustic - p'

Related by linearized R-H conditions

Kovásznay (1953)

Ribner (1954)

Comparison

Direct numerical simulation

Re_λ	M	M_t
39	1.05	0.05
38	1.28	0.15
39	1.28	0.22
38	1.28	0.26
38	1.28	0.31
38	1.50	0.15
39	1.50	0.22
39	1.51	0.31
39	1.51	0.37
39	1.87	0.22
39	1.87	0.31
40	2.50	0.22
40	3.50	0.16
41	3.50	0.23
42	4.70	0.23
42	6.00	0.23
73	1.50	0.14
73	1.50	0.22
72	1.52	0.38
74	3.50	0.15

J. Fluid Mech. (2013), vol. 717, pp. 293–321. © Cambridge University Press 2013
doi:10.1017/jfm.2012.573

293

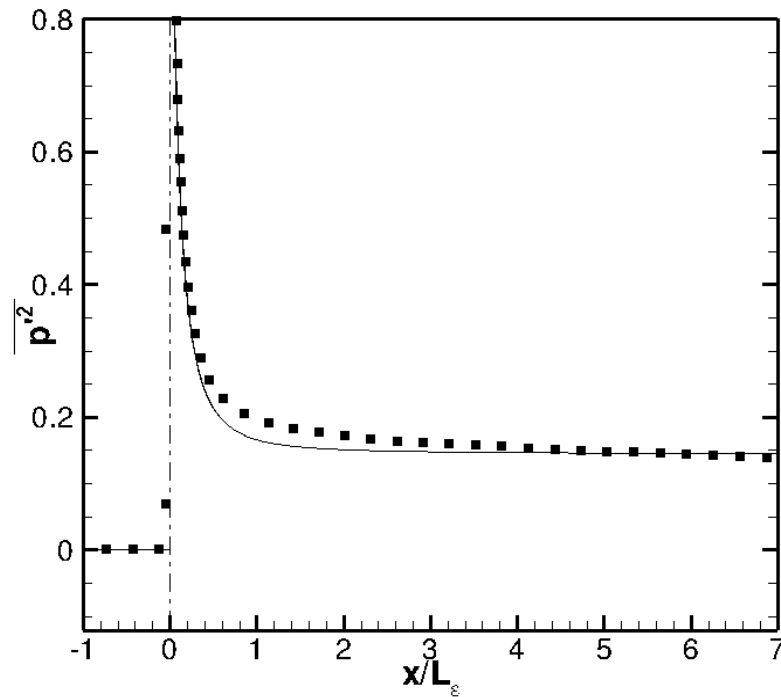
Reynolds- and Mach-number effects in canonical shock–turbulence interaction

Johan Larsson†, Ivan Bermejo-Moreno and Sanjiva K. Lele

Low M_t cases used to compare with theory

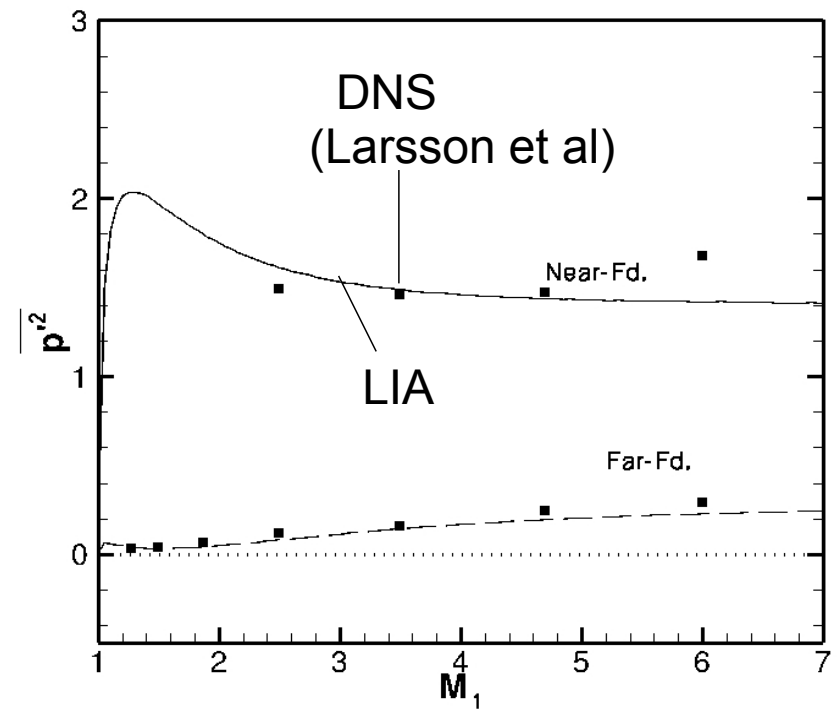
Acoustic Mode

Acoustic mode



$M = 3.50$, DNS (symbol) LIA (line)

- High Near-field value
- Exponential decay to a finite far-field value

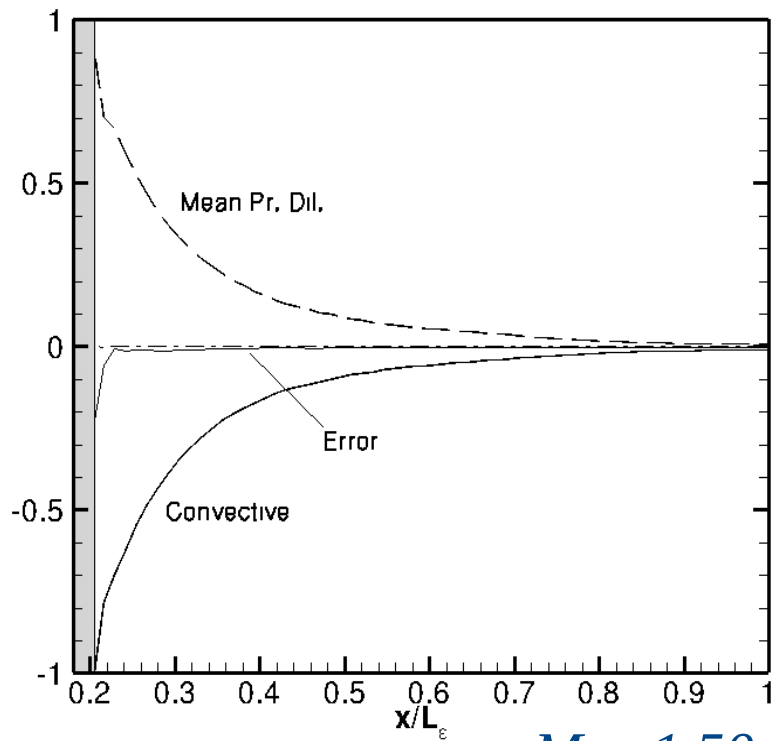


Budget for acoustic mode

$$\underbrace{\frac{\partial}{\partial x} \left(\frac{\overline{p'^2}}{2} \right)}_{\text{Convective}} + \underbrace{\frac{2\gamma\bar{p}}{\tilde{u}} \overline{p' \frac{\partial u_j''}{\partial x_j}}}_{\text{Mean Pr. Dil.}} + \text{Other Terms} = 0$$

Budget computed behind the shock

Friedrich et al. (1999)



$M = 1.50$

Linear Theory

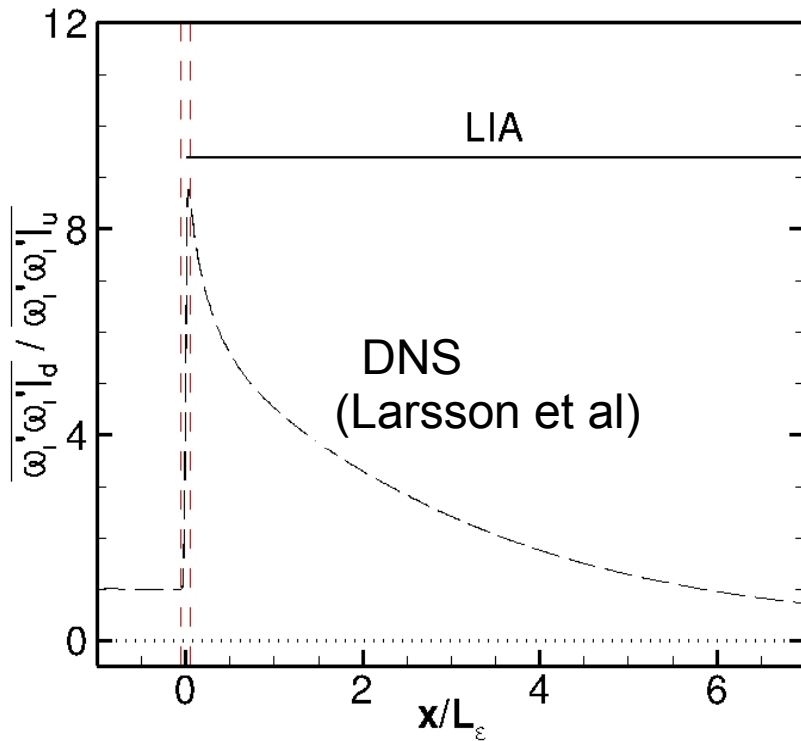


$$\frac{\partial}{\partial x} \left(\frac{\overline{p'^2}}{2} \right) = - \frac{2\gamma\bar{p}}{\tilde{u}} \overline{p' \frac{\partial u_j'}{\partial x_j}}$$

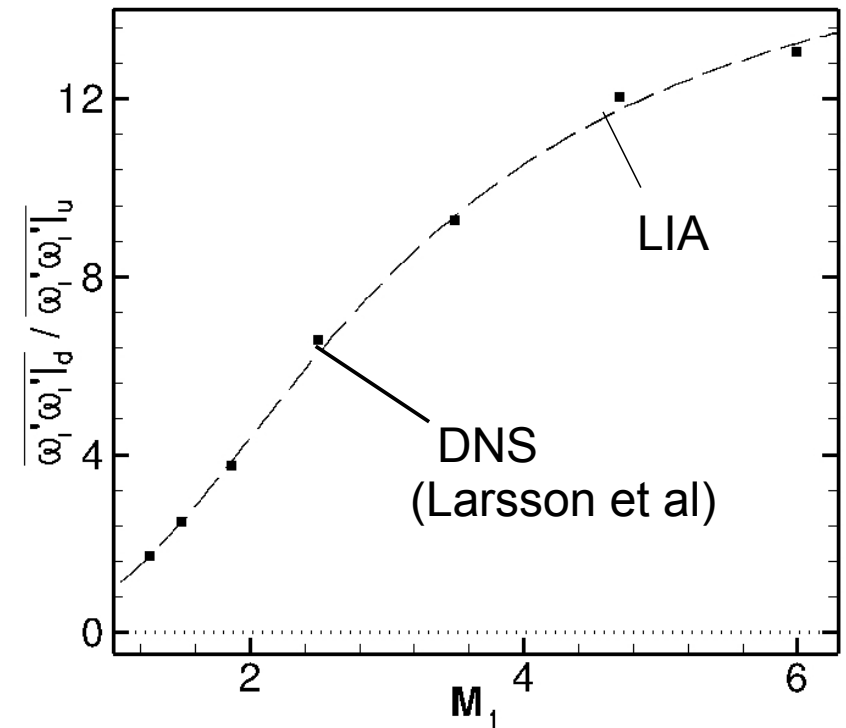
LIA captures acoustic mode well

Vorticity mode

Vorticity mode



$M = 3.50$



Near field : DNS (symbol) LIA (line)

➤ LIA : Const. Vorticity variance

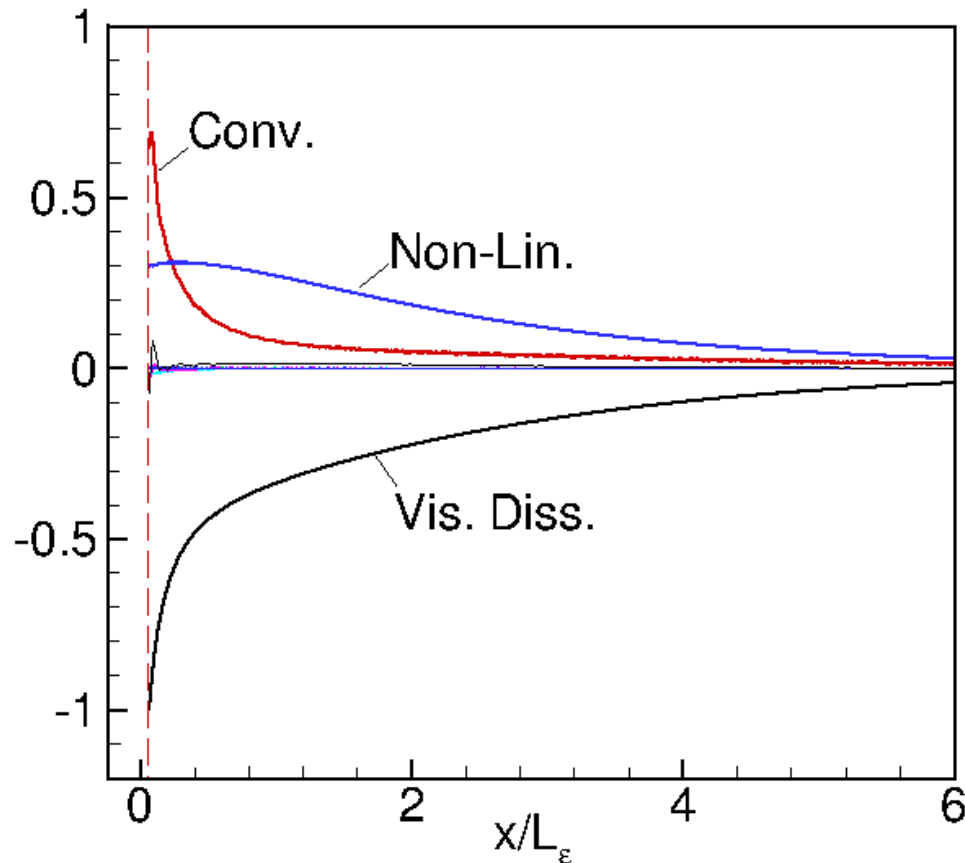
➤ DNS : Downstream decay

$$\frac{\partial}{\partial x} \left(\frac{\overline{\omega' \omega'}}{2} \right) = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\overline{\omega' \omega'}}{2} \right) = ?$$

Budget for vorticity mode

$$\overline{u_k \partial_k \omega'_\alpha \omega'_\alpha} = \underbrace{\overline{2\omega'_\alpha \omega_\alpha \partial_k u'_\alpha}}_{\text{Stretching}} - \underbrace{2\epsilon_{\alpha j k} \overline{\frac{1}{\rho} \frac{\partial \omega'_\alpha}{\partial x_j} \frac{\partial \sigma_{km}}{\partial x_m}}}_{\text{Viscous Diss.}} + \text{Other terms}$$

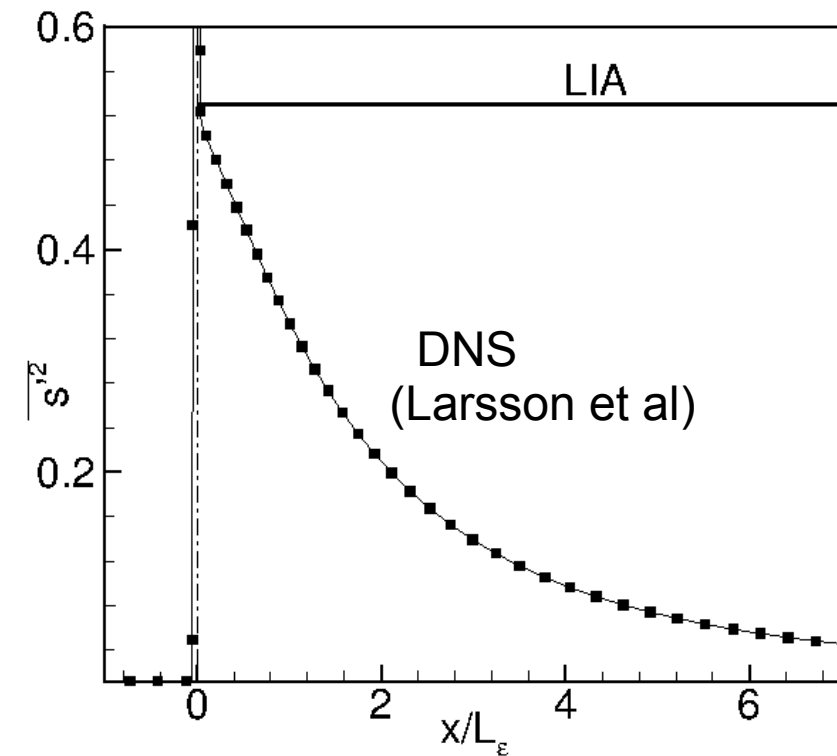


- Non-linear Term : Positive contribution
- Viscosity : Negative contribution

LIA cannot capture vorticity mode

Entropy mode

Entropy mode



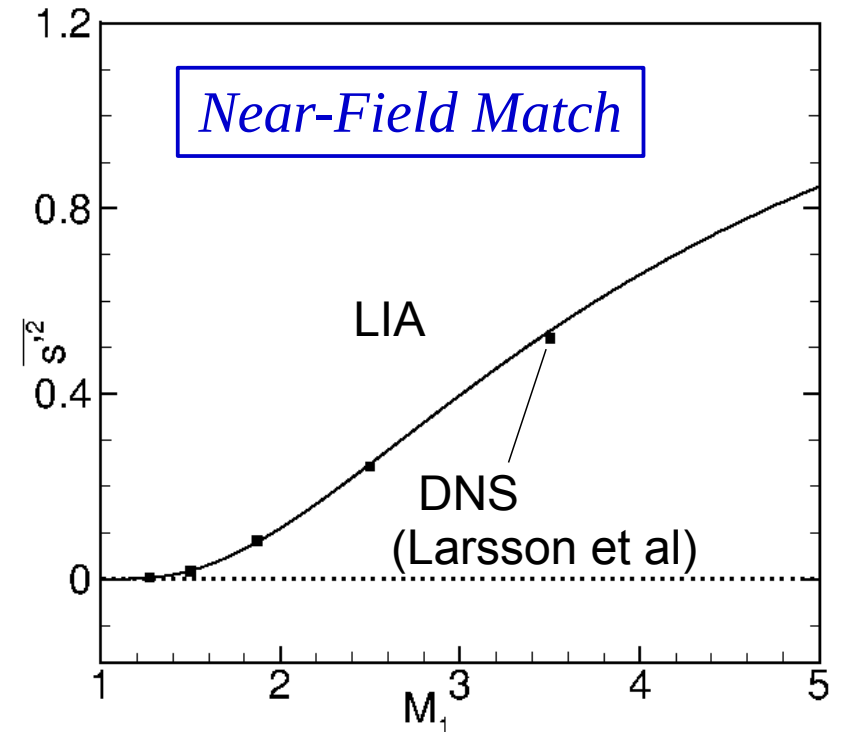
$M = 3.50$

➤ LIA : Const. entropy variance

➤ DNS : Downstream decay

$$\frac{\partial \overline{s'^2}}{\partial x} = 0$$

$$\frac{\partial \overline{s'^2}}{\partial x} = ?$$

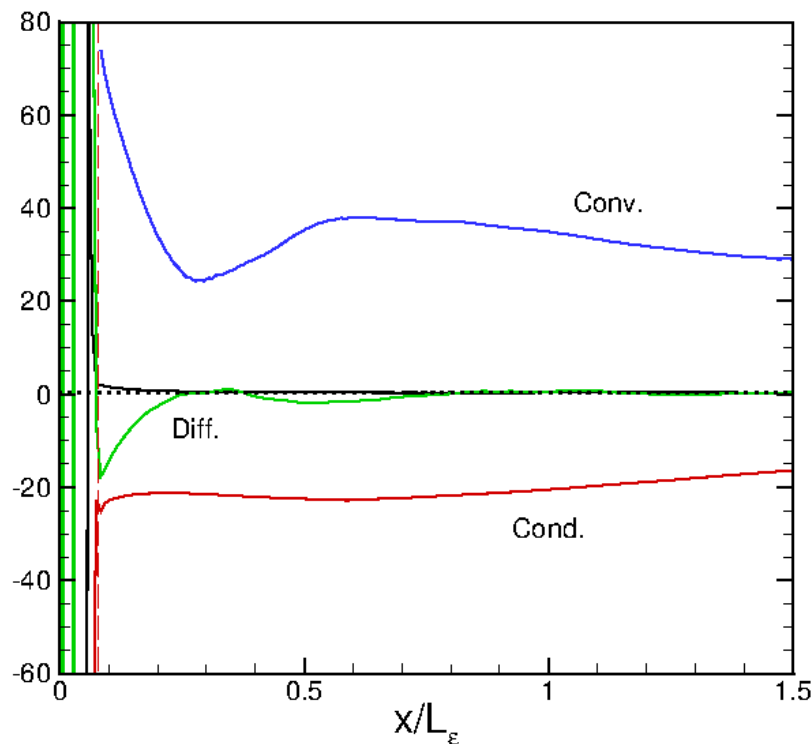


Near field : DNS (symbol) LIA (line)

Budget for entropy mode

$$\overline{\rho \tilde{u}_k \partial_k s'^2} = \underbrace{\partial_1 \overline{\rho u'_1 s'^2}}_{\text{Diffusion}} - \underbrace{\frac{s'}{T} \partial_k q_k}_{\text{Conduction}} + \underbrace{2 \frac{s'}{T} \sigma_{ik} \partial_k u_i}_{\text{Viscous}} + \text{Other terms}$$

Gerolymos and Vallet (2013)



- Diffusion : Negative contribution
- Conduction : Negative contribution
- Viscosity : Small Positive contribution

*LIA cannot capture entropy mode**

** - under-resolved post-processing*

Summary

Good match in the near-field for all three modes

Good match in the far-field as well for the acoustic mode

Mismatch in the far-field for the vorticity and entropy modes

Thank you for listening !

Extra slides

Application to shock turbulence interaction

➤ 2D single mode plane wave interaction $e^{ik(mx+ly-u_1mt)}$

➤ Governing equations : Linearized Euler equations

$$\rho'_t + \bar{u}\rho'_x = -\bar{\rho}(u'_x + v'_y) \quad s'_t + \bar{u}s'_x = 0$$

$$u'_t + \bar{u}u'_x = -\frac{1}{\bar{\rho}}p'_x \quad v'_t + \bar{u}v'_x = -\frac{1}{\bar{\rho}}p'_y$$

$$\frac{p'}{\bar{P}} = \frac{\rho'}{\bar{\rho}} + \frac{T'}{\bar{T}}$$

$$\frac{s'}{C_p} = \frac{p'}{\gamma\bar{P}} - \frac{\rho'}{\bar{\rho}}$$

➤ Boundary conditions :
Linearized Rankine Hugoniot

$$\frac{u'_2 - \xi_t}{\bar{u}_1} = \frac{(\gamma - 1)M_1^2 - 2}{(\gamma + 1)M_1^2} \left(\frac{u'_1 - \xi_t}{\bar{u}_1} \right) + \frac{2}{(\gamma + 1)M_1^2} \left(\frac{T'_1}{\bar{T}_1} \right)$$

$$\frac{v'_2}{\bar{u}_1} = \frac{v'_1}{\bar{u}_1} + \frac{2(M_1^2 - 1)}{(\gamma + 1)M_1^2} \xi_y$$

$$\frac{p'_2}{\bar{p}_2} = \frac{4\gamma M_1^2}{2\gamma M_1^2 - (\gamma - 1)} \left(\frac{u'_1 - \xi_t}{\bar{u}_1} \right) - \frac{2\gamma M_1^2}{2\gamma M_1^2 - (\gamma - 1)} \left(\frac{T'_1}{\bar{T}_1} \right)$$

$$\frac{\rho'_2}{\bar{\rho}_2} = \frac{4}{(\gamma - 1)M_1^2 + 2} \left(\frac{u'_1 - \xi_t}{\bar{u}_1} \right) - \frac{(\gamma - 1)M_1^2 + 4}{(\gamma - 1)M_1^2 + 2} \left(\frac{T'_1}{\bar{T}_1} \right)$$